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ABSTRACT

A research project developed a computer program for analyzing time series quasi-experimental data. The program generates a nonstationary, integrated moving average time series model; it is used to estimate a parameter which indexes the instantaneous change in level of the time series due to a predesignated treatment. The entire method is based on a Bayesian model for which a uniform rectangular prior to and following the treatment carry primary weights. Computer Print-out Appendix deleted from document due to irreproducibility of original. (Author)

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NOTES ON TIME SERIES ANALYSIS

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ABSTRACT

This paper describes a program which was developed for analyzing time series quasi-experimental data. The computer program outlined generates a non-stationary, moving average, time series model which indexes the instantaneous change in level of the time series due to a predesignated treatment. The observations just prior to and just following the treatment are given primary weight in the model.

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SPECIAL REPORT No. 722
COMPUTER-BASED PROJECT for the EVALUATION of MEDIA for the HANDICAPPED

Title: NOTES ON TIME SERIES ANALYSIS

BY: Robert Pruzek & Jack H. Bond

BACKGROUND

The Computer Based Project for the Evaluation of Media for the Handicapped, based on contract #OEC-9-423617-4357 (616) between the Syracuse (N.Y.) City School District and the Media Services and Captioned Films Branch, Bureau of Education for the Handicapped (United States Office of Education) for the five year period July 1, 1969 through June 30, 1974. The major goal is to improve the instruction of handicapped children through the development and use of an evaluation system to measure the instructional effectiveness of films and other materials with educable mentally handicapped (EMH) children, in-service training and media support for special teachers, and studies related to the evaluation process and the populations used.

The Project has concentrated on the 600 films and 200 filmstrips from the Media Services and Captioned Films (BEH - USOE) depository; however, specific packages from Project LIFE, various elementary math curricula, and selected programs from Children's TV Workshop have also been evaluated. The evaluation model used requires that: 1) objectives of materials be specified and written; 2) instruments be constructed to test and measure effectiveness; and, 3) children be the major sources of evaluation information. A number of instruments and methodologies are employed in the gathering of cognitive and affective data from 900 EMH children and 80 special teachers to make the effectiveness decisions. Over half of the EMH population can neither read or write; therefore, a unique Student Response System (SRS) is employed, consisting of a twenty station G.E.-1000 SRS which can be operated in a group or individual recording mode and is connected to a remote computer system. The computer capabilities consist of remote telephone connections to the Rome (N.Y.) Air Development Command, the Honeywell time-shared network, and the Schenectady (N.Y.) G E Research and Development Center; and batch mode capabilities of the Syracuse City Schools, Syracuse University, and various commercial sources.

In-service and media support activities provide on-the-job training for teachers, teacher aides, equipment, and materials to the special teachers in the city schools. The research activities have centered around investigations and special problems related to the development of the evaluation model. The four major areas considered are: 1) testing effects, 2) captioning effects, 3) special student characteristics; and, 4) evaluation procedures validation.

Documentation of the major activities appear in the five annual reports and the 600 evaluations prepared on materials used. Staff members were encouraged to prepare special reports and the attached paper is one of these. The opinions expressed in this publication do not necessarily reflect the position or policy of the Computer Based Project, the United States Office of Education, or the Syracuse City School District, and no official endorsement by any of the agencies should be inferred.

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Notes on Time Series Analysis

These notes are to describe a program for analysing time series quasi-experimental data. The method of time series analysis is called an integrated, moving average method. It has been found to be useful in a wide variety of industrial and economic applications and seems to hold considerable potential for studying educational data. The primary reference is George Box and George Tiao - " A Change in Level of a Non-stationary time series", Biometrika, 1965, p. 181-192. The assumptions underling the Box-Tiao method are somewhat difficult to describe in words in their entirety, but three major points should be made:

1) There will be an absence of cycles or systematic trends in the original data across the entire series. That is, the data should appear to fluctuate more or less randomly around a general line of trend for the entire series. Sustained drifts in one direction or another will tend to invalidate this model.

2) The Correlogram of the original data is to be free of systematical cycles; it should show more-or-less random fluctuation around the base line. In particular, the correlogram should not show the damped, cyclic curve characteristic of autoregressive time series. In fact Box-Tiao offered their moving average methology largely to avoid major problems of auto-regressive time series methods.

3) The correlogram of the differences between successive observations in the time series has a "lag-one" correlation which is large in absolute value (near .50 usually) whereas, all higher lag correlations - 2,3,4 etc. - are to be near zero. (The latter is a statical question which may be very much open to discussion in the context of a particular experiment.)

Further general points to be made are that the time series observations are assumed to have been made at equally spaced time intervals; and that there is interest in testing the hypothesis of a possible shift in the level of the series which is associated with the occurance of a particular event called a treatment. The Box-Tiao model is properly described as a non-stationary,

integrated, moving average time series process. In its simplest form (not one to be described except briefly here), the model says that an observation at any particular point t, in the time series, is a sum of three terms: (1) a fixed but unknown location parameter, L, (2) the product of a quantity gamma(γ) times the accumulation of a previous set of α values up to any point t in the series, and (3) the alpha (α_t) value associated with that particular point in time. The alphas are taken as independent random normal deviates, which are assumed to have constant variance of sigma squared for each point of the series.

We shall consider a slightly more complex form of the model, in which three major parameters are involved. Two equations may initially be used to describe the model on which the program is based. The first, which is equivalent to the one in the preceeding paragraph is that for any one of the observations Z_t prior to treatment T, any value

$$Z_t = L + \gamma \sum_{j=1}^{t-1} \alpha_{t-j} + \alpha t \qquad (1)$$

where L is the fixed location parameter, αj is a random (normal) deviate associated with the jth point in time and γ is a parameter indexing the proportion of the assumulated α s ($\sum_{j=1}^{t-1} \alpha_{t-j}$) which have been absorbed into the time series itself.

The second equation states that for any observation Z_t at time t following the predesignated treatment, the observation is

$$Z_t = L + \delta + \gamma \sum_{j=1}^{t-1} \alpha_{t-j} + \alpha_t \qquad (2)$$

This equation differs from (1) above only by the addition of the parameter δ (delta) which indexes an instantaneous shift in level of the time series which is due to treatment T. The program, however, involves a "reparameterization"

stating that $a_{t-j} + \mu + a_{t-j}$ thus allowing for a general drift or slope, of the time series -- i.e. other than horizontal.

This has the effect of modifying equation (2) to

$$Z_t = L + \delta + (1 + \mu(t-1)) + \gamma \sum_{j=1}^{t-1} a_{t-j} + a_t \quad (2a)$$

where the series is expected to have drifted a total of $(1 + \mu(t-1))$ units at time t. The three parameters of interest then became L, μ & δ.

The Program and its outputs:

While there are, in fact, two programs which were written by Gene Glass and Thomas Maguire, for Box-Tiao method of times series analysis, the second program has been found to yield uninterpretable results with artificial data sets which were intended to be relatively straight-forward in their characteristics. We shall ignore the second program until a later date. For now we turn to a description of the working computer program. The data are assumed to be gathered in a running time series, with a predesignated event T which is specified to have occured during the course of the series. Any input data are required to be a running series of $N_1 + N_2$ observations. I shall describe the outputs consecutively with respect to the labelled output listing.

For artificial data with $N_1 = 26$ and $N_2 = 9$, Output I is simply a listing of the input itself. A table like that of Table IA ought to be inspected for each data set simply to get a general impression of the nature of the time-trends. Output II is the table of product-moment correlation coefficients of lag 1, 2, ..., 3/4 N_1 for the pre-T. observations; their values are plotted automatically in Figure IIA. This correlogram (IIA) should be checked to see that there are no systematic trends, or obvious cycles in the time series. In particular, IIA should not show a conventional damped sine curve, which tends to show up when an

autoregressive model is appropriate. Table III and Figure IIIA are directly analogous to II and IIA and the same type of inspection is recommended.

The outputs (IV, IVA) and (V, VA) are lag correlations for pre - and post - treatment differences. In the case of Table IV, the lag-one correlation should be fairly large in absolute value (often near .50) and the succeeding correlations should tend to approximate zero.

The remaining tables and figures require a brief preamble. The Box-Tiao method is initially based on a Bayesian method of analysis which assumed that a parameter labelled gamma (γ) was initially known from previous research in the particular area where the time series data has been gathered. The latter part of the Box-Tiao paper, however, indicates when this value of gamma is not known, the sample data may be used to estimate gamma by generating a maximum likelihood solution taken over all possible values of gamma under the assumption that the prior for γ is rectangular. This is what is done in the Glass-Maguire computer programs. The maximum likelihood statistics for L, μ and ζ correspond to the row of Table VI where the posterior of γ is largest. The delta parameter δ , or its estimate, is the primary focus of analysis since this is the measure of instantaneous change in level of the time series. In addition to the 200 values of delta hat ($\hat{\delta}$) which are given over the corresponding range of gammas, we also print the 200 values of mu hat ($\hat{\mu}$) and $\hat{L}$. These, respectively, estimate the general rate of ascent or descent (assuming the slope is fixed) and the overall location parameter.

The major focus of interpretation should usually be Figure IVA in combination with Table VI. In particular, the plot of "X" in Figure VIA gives immediate graphical evidence of the maximum likelihood value of γ based on the line for $\gamma = \gamma_{ML}$ (.48 for the artificial data set), where the specific values of $\hat{L}$, $\hat{\mu}$ and $\hat{\delta}$ are printed as well as the

standard error estimates $\hat{\sigma}_{\hat{\mu}}$ and $\hat{\sigma}_{\hat{\delta}}$ and the t- statistics for $\hat{\mu}$ and $\hat{\delta}$. The t-values for $\hat{\delta}$ are plotted for all values of γ in Figure VIA as the "0" values. The object is to find statistically significant values of the instantaneous effect δ . It should be noted that δ represents the difference between two exponentially weighted means; obs. immediately preceding, and following, the event T are weighted more heavily than their adjacent (preceding, or following) observations. Increasing the number of observations either prior to, or after, the treatment event serves largely to increase the degrees of freedom for the t-statistic (which is $N_1 + N_2 - 3$) as well as to stabilize the estimates of L, μ and δ . Outputs VII and VIIA are essentially "frills" in that they present no fundamentally new information beyond that given in Table VI. Nevertheless, the visual impression of; variability of the statistic $\hat{\delta}$ over the 200 values of γ , as well as its confidence interval, may facilitate speedy interpretation and comparisons of several different data sets.

The statistics $\hat{L}$ and $\hat{\mu}$ may be somewhat difficult to interpret, until one recognizes that they are not straight-forward unweighted measures; in particular, $\hat{L}$ is properly thought of as an estimate of overall <u>level</u> of the series, with primary emphasis on scores early in the series, and $\hat{\mu}$ is an estimator of slope which is more heavily weighted by observations toward the end of the series. Nevertheless, the statistics should be reasonably comparable from one data set to another at least when the sums N_1 N_2 do not differ readically. The values for $\gamma_{ML} = .48$ of $\hat{L} = 4.82$ and $\hat{\mu} = .404$ do seem to correspond crudely to overall <u>level</u> and <u>slope</u> respectively, given the above qualifications.

To summarize the major points, we have a non-stationary, moving average, time series model, a computer program for which has been generated especially to estimate a parameter δ , which indexes the instantaneous change in

level of the time series due to a predesignated treatment. The observations just prior to and just following the treatment are given primary weight in the model. The method is based on a Bayesian model; a uniform rectangular prior distribution is assumed in the program for the unknown parameter gamma. A maximum likelihood estimate of that gamma is printed and plotted.

Finally, it may be useful to note that the above statistics, together with the estimated residual variances (1st columns of Output VI), may be used to make planned (or even post- hoc) comparisons of comparable statistics for any comparable sets of time series points. For instance, see the chapter on planned comparisons in Hays (1963).

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